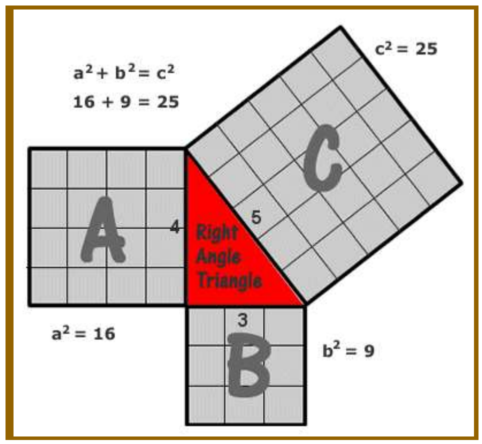
Geometry

"Let no man ignorant of geometry enter here."
Inscribed above the door - Plato's Academy in Athens

Pythagoras and his school - as well as a handful of other mathematicians of ancient Greece - was largely responsible for introducing a more rigorous mathematics than what had gone before, building from first principles using axioms and logic. Before Pythagoras, for example, geometry had been merely a collection of rules derived by empirical measurement. Pythagoras discovered that a complete system of mathematics could be constructed, where geometric elements corresponded with numbers, and where integers and their ratios were all that was necessary to establish an entire system of logic and truth.
History records that Pythagoras and Diophantus were probably the two most well known mathematicians that had anything to do with right triangles with integer sides. The famous Pythagorean Theorem states that in any right-angled triangle, the sum of the squares of the two legs is equal to the square of the hypotenuse. Another way of stating it is that the area of the square constructed on the long side of a right triangle is equal to the area of the two squares created on the two shorter sides. Written as an equation:

$$a^2 + b^2 = c^2.$$

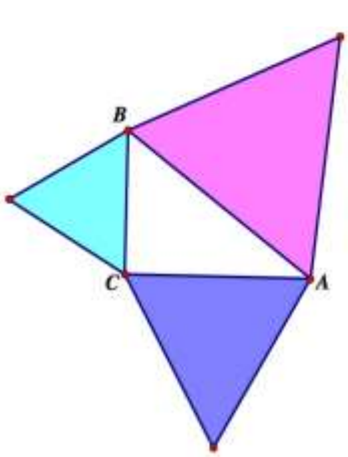
The simplest and most commonly quoted example of a Pythagorean triangle is one with sides of 3, 4 and 5 units ($3^2 + 4^2 = 5^2$), as can be seen by drawing a grid of unit squares on each side as in the diagram at right), but there are a potentially infinite number of other integer "Pythagorean Triples", starting with (5, 12 13), (6, 8, 10), (7, 24, 25), (8, 15, 17), (9, 40, 41). There are an infinite number without any proportional relationship. Note that (6, 8, 10) is not what is known as a "primitive" Pythagorean triple, because it is just a multiple of (3, 4, 5).
The 3-4-5 right triangle proportions, though not the mathematics, were well known to the medieval master masons and could be used for constructing right angles, however they could also layout right angles, 45 degree, and 30 degree angles (plus double and halves) with compasses and a straight edge.

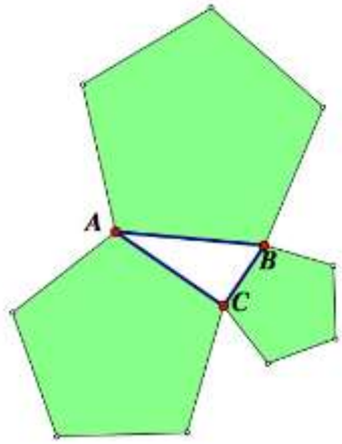


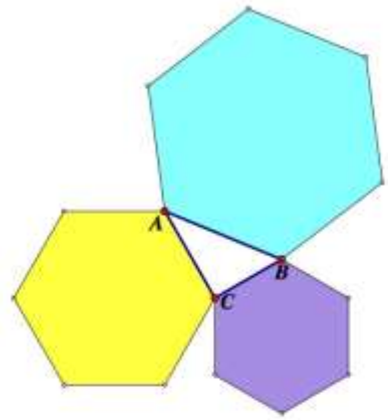
Pythagoras' (Pythagorean) Theorem

Pythagoras' Theorem and the properties of right-angled triangles seems to be the most ancient and widespread mathematical development after basic arithmetic and geometry, and it was touched on in some of the most ancient mathematical texts from Babylon and Egypt, dating from over a thousand years earlier.

What about other shapes? Triangles? Rectangles? Pentagons? Hexagons? You may want to explore this for yourself.







The area of the polygon on the hypotenuse equals the sum of the polygon areas shown on the sides.

Some Pythagorean Theorem Trivia

Ancient Greek mathematicians, and especially the followers of Pythagoras and his school, were entranced by numbers which could be made up by arranging points in regular patterns on a plane or in space. The simplest such figure with three equal angles and three equal sides, is the equilateral triangle; often seen displayed in the Scottish Rite degrees:

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*    *     *      *       *
    * *   * *    * *     * *
         * * *  * * *   * * *
               * * * * * * * *
                      * * * * *
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Using it as a basic building block in the following manner; we find that it generates the numbers 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66 … which are consequently referred to as the *triangular* numbers.

1	3	6	10	15	21	28	36	45	55
66	78	91	105	120	136	153	171	190	210
231	253	276	300	325	351	378	406	435	465
496	528	561	595	630	666	703	741	780	820
861	903	946	990	1035	1081	1128	1176	1225	1275
1326	1378	1431	1485	1540	1596	1653	1711	1770	1830
1891	1953	2016	2080	2145	2211	2278	2346	2415	2485
2556	2628	2701	2775	2850	2926	3003	3081	3160	3240
3321	3403	3486	3570	3655	3741	3828	3916	4005	4095
4186	4278	4371	4465	4560	4656	4753	4851	4950	5050

Triangular Numbers.

Triangular numbers and the square numbers are related in a simple manner. It is that the sum of any two consecutive triangular numbers is always a square number and, moreover, all square numbers can be formed in this way. For example

$1+3 = 2^2$

$3+6 = 3^2$

$6+10 = 4^2$

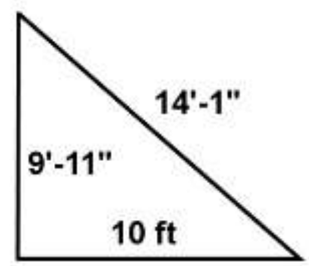
$10+15 = 5^2$

and so on. These relationships were known to the early Greeks.

Triangular numbers often appear in the most unexpected places. All perfect numbers are known to be triangular. This means, among other things, that a perfect number of pool or billiard balls can always be ‘racked up’ in a triangle of a suitable size; even if we have 8128 of them. Triangular-square numbers turn out to be useful. They can be used to generate right-angled (or Pythagorean) triangles of a particular kind; namely those in which the two shorter sides differ in length from each other by a single unit. The smallest such triangle is the well-known 3, 4, 5 one. The next smallest has side lengths 20, 21, 29; i.e. $20^2 + 21^2 = 29^2$, comprised of consecutive triangular numbers:
(190+210) + (210+231) = (406+435).

Perfect Numbers:
In number theory, a perfect number is a positive integer that is equal to the sum of its proper positive divisors. The first perfect number is 6, because 1, 2, and 3 are its proper positive divisors, and 1 + 2 + 3 = 6. The next perfect number is 28 = 1 + 2 + 4 + 7 + 14. This is followed by the perfect numbers 496 and 8128.

The next triple with sides: 119, 120, 169 has some interest. In the English units of measurement, these numbers can be labeled with inches. Thus the sides of the right triangle would be 9 ft.-11in. , 10 ft, and 14ft.'-1in. The perimeter would be 34 ft. A chain of 34 feet length marked with adjoining 9'-11 and 10' divisions would be useful for squaring corners of a fence or a building foundation once pulled taut on the 14'-1" hypotenuse. The fact that the perimeter is one foot beyond 33 is interesting but has no known Scottish Rite numerological history.



34 ft chain

More interesting geometric symbols are discussed in the Scottish Rite 27th degree and in Albert Pike's book "Esterica".